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An Analysis of Shareholder Agreements

Abstract

Shareholder agreements govern the relations among shareholders in privately-held companies, such as joint ventures or venture capital-backed firms. We provide an economic explanation for the use of put and call options, pre-emption rights, catch-up clauses, drag-along rights, demand rights, and tag-along rights in shareholder agreements. We view these clauses as a response to a problem of dynamic, double moral hazard, whereby the value of the venture depends on ex ante investments and ex post transfers. Contract clauses i) preserve the incentives to make ex ante investments and ii) minimize ex post transfers. We extend our framework to discuss the use of other clauses, such as the option to extend the life of a business alliance. (JEL: G34).

Keywords: Shareholder Agreements; Put Options; Call Options; Pre-emption Rights; Catch-up Clauses; Drag-along Rights; Demand Rights; Tag-along Rights.
1 Introduction

Shareholder agreements specify the rights and duties of shareholders when those prescribed by law and regulation are thought not to be appropriate. Shareholder agreements mostly govern private companies. Prominent examples are joint venture contracts and agreements between venture capitalists and entrepreneurs. In addition to specifying the shareholders’ rights and duties, shareholder agreements mandate a dispute resolution process, allocate various types of options, and impose restrictions on the transfer of shares.1

Shareholder agreements generally grant the parties the following rights: the option to put their stakes to their partners or to call their partners’ stakes, in part or in whole, at a strike price that is typically equal to ‘fair’ value; pre-emption rights that confer precedence to the parties in buying their partners’ stakes at ‘fair’ value in case the partners should wish to exit the venture; catch-up clauses that maintain the parties’ claims to part of the payoff from a trade sale or an IPO when the parties have ceded their stakes to their partners following the partners’ exercise of a call option; drag-along rights that allow the parties to force their partners to join them in selling their stakes to a trade buyer in the case of a trade sale; demand rights that allow the parties to force their partners to agree to taking the venture public in an IPO; and tag-along rights (or piggy-back rights, or co-sale agreements) that allow the parties to demand of a trade buyer buying their partners’ stakes the same treatment as received by their partners.2

We view the preceding clauses as serving to i) preserve the incentives to make ex ante investments when ex post renegotiation is possible and ii) minimize ex post transfers in a setting of dynamic, double moral hazard. Ex post renegotiation is to be avoided because it distorts ex ante investment incentives. Opportunities for renegotiation arise when it is desirable to minimize value-reducing

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1 See Appendix 1. Standard shareholder agreements are described in Bernstein (1988), Freedman (1994), Martel (1991), and Stedman and Jones (1990). Joint venture and venture capital contracts are special cases of shareholder agreements. The former are described in Herzfeld and Wilson (1996), Linklaters et al. (1990), and Scott (1999). The latter are described in Bartlett (1994) and Stedman and Jones (1990). Contracts appear to be strikingly similar across countries and legal systems (Martel, 1991).

2 Note that drag-along rights may conflict with pre-emption rights: the parties that can pre-empt their partners cannot be dragged-along, and those that can be dragged-along cannot pre-empt their partners.
transfers ex post, and when the venture is to be sold to a trade buyer or to be taken public in an IPO.

We show that put and call options alleviate the problem of value-decreasing transfers from the joint enterprise. Pre-emption rights and tag-along rights deny the parties the ability to impose renegotiation by threatening the sale of their stake to a trade buyer who would decrease the value of the venture, or to conspire with a trade buyer who would increase the value of the venture to exclude their partners from sharing in that increase in value. Catch-up clauses deny the parties holding a call option the ability to profit from exercising their call prior to a trade sale or an IPO. Drag-along rights (respectively, demand rights) deny the parties the ability to impose renegotiation by vetoing or refusing to take part in a value-increasing sale to a trade buyer (respectively, IPO).


³Call options that are embedded in convertible preferred stock constitute an exception. In this paper, we do not attempt to address the optimality of financial clauses (see Aghion, Bolton and Tirole (2001) on the optimality of one restricted type of contract in venture capital). Instead, we take the contracts as given and consider their effect on ex ante investments and ex post transfers.
We proceed as follows. We present the initial setting in Section 2. We analyze the case where the venture must remain the property of one or both founding parties in Section 3. We analyze the case in which the venture must be sold to a trade buyer or taken public in an IPO in Section 4. We briefly discuss continuation and termination in alliances in Section 5. Section 6 concludes. Appendix 1 contains a brief overview of the clauses found in standard shareholder agreements. Appendix 2 contains most proofs.

2 The initial setting

Two parties $a$ and $b$ jointly undertake a venture.

Each party must make an investment towards the success of the venture. Let $i_i$ denote the investment made by party $i$, $i \in \{a, b\}$, at a cost $i_i$.

Once undertaken, the venture can be put to four uses. It can remain a joint enterprise; be acquired by one or the other founding party in its entirety; or be acquired in whole or in part by a trade buyer.\footnote{The venture can also taken public in an IPO. As this outcome can be viewed as very similar to a trade sale (see Section 4.2), we do not analyze it separately.} We denote $u$ the use to which the venture is put, $u \in \{a b, a, b, t b\}$.

The value of the venture in use $u$ is $V_u(i_ai_b, t_i + t_j, s)$. In addition to being affected by the investments $i_a$ and $i_b$, the value of the venture is also affected by the transfers $t_i$ and $t_j$ in which the parties to the venture may engage.\footnote{The multiplicative formulation for investments is intended to capture their strong complementarity, whereas the additive formulation for transfers reflects their substitutability. Note that it is only the investments made by the founding parties that affect the value of the venture, even if the venture is later sold to a trade buyer.} Any party to the venture, whether a founding party or a trade buyer who has acquired the stake of a founding party, may engage in a transfer. Thus, $i, j \in \{a, b, tb\}$. The personal benefit to party $i$ of engaging in a transfer $t_i$ is $B_i(t_i) = \alpha_i B(t_i)$, with $\alpha_i \geq 0$ an index of the relative importance of $i$'s personal benefit. We assume $\alpha_a > \alpha_b$.

Finally, the value of the venture depends on the state of the world $s$. We consider two states: the state $s_{tb}$, in which a trade buyer who can increase the value of the venture offers to buy the
venture, and the state $s_{tb}$ in which there is no such trade buyer. Thus:

$$V_{tb}(I, T, s_{tb}) > V_u(I, T, s_{tb})$$

for $u \neq tb$, $I \equiv i_a i_b$, and $T \equiv t_i + t_j$. We leave open the possibility that a trade buyer who cannot increase the value of the venture exists in state $s_{tb}$. In either case, we assume that a trade buyer has no bargaining power when bargaining with one or both founding parties.

We note that the investments $i_a$ and $i_b$ are made before the state $s$ is realized, whereas the transfers $t_i$ and $t_j$, $i, j \in \{a, b, tb\}$, are made after the state is realized. Thus, transfers are made from the venture’s payoff rather than the investments.

We make the following assumptions: $V_{u,1} > 0$, $V_{u,11} < 0$, $V_{u,2} < 0$, $V_{u,22} < 0$, $V_{u,12} < 0$, $B' > 0$, $B'' = cst < 0$. These assumptions imply that the value of the venture is increasing and concave in investment, that it is decreasing and concave in transfers, that transfers decrease investment, and that the personal benefits to transfers are increasing and concave in transfers. The assumption that $B''$ is constant simplifies the comparative statics analysis.

We also assume that $V_{u,2}(I, 0, s) + B'(0) < 0$. This implies that no transfer will take place when the venture has a single owner. A party owning only part of the venture may, however, wish to engage in a transfer. This is because the cost of the transfer is shared with the other party in proportion to each party’s stake, whereas the benefit of the transfer is received in its entirety by the party engaging in the transfer. We assume that transfers do occur when the venture has more than a single owner.

Let $\gamma$, $0 < \gamma < 1$, denote party $a$’s initial stake in the venture. We first show that — regardless of the use $u$ to which the venture is put, and the realized state $s$ — the value of $\gamma$ that induces the founding parties $a$ and $b$ to make the investments that maximize the value of the venture, subject to the constraint imposed by the problem of double moral hazard, but absent the transfer problem,
is $\gamma = \frac{1}{2}$.

**Proposition 1** The value of the venture $V_u(i_a, i_b, 0, s) - i_a - i_b$ is maximized at $\gamma = \frac{1}{2}$.

**Proof:** See Appendix 2.

Proposition 1 implies that the founding parties’ payoffs should always be in the proportions $\gamma = \frac{1}{2} = 1 - \gamma$. This is a simple consequence of the symmetry of the parties. These proportions should not be renegotiated, for renegotiation would distort the value of the investments made by the founding parties. It would thereby decrease the value of the venture.

However, when taking the transfer problem into account, incentives to renegotiate will arise ex post. This is because partners have different abilities to transfer value. Incentives to renegotiate also arise when a trade buyer appears who can increase the value of the venture, for a founding party may then seek to extract more than his share of the increase in value by threatening to hold up the sale.

Even in the case where the venture should remain the joint property of the two founding parties, the value of the venture can be increased ex post by changing the parties’ stakes from the values $(\gamma, 1 - \gamma)$ to the values $(\gamma', 1 - \gamma')$ that minimize ex post transfers. To see this, recall that party $a$ derives a greater personal benefit from a transfer than does party $b$ ($\alpha_a > \alpha_b$). Proposition 2 shows that $a$’s stake $\gamma'$ should be reduced below $\frac{1}{2} = \gamma$.

**Proposition 2** Following the making of the investments $i_a$ and $i_b$ and the realization of the state $s_{ib}$, the stake that maximizes the value of the venture when it remains the joint property of the two founding parties is $\gamma' < \frac{1}{2} = \gamma$.

**Proof:** See Appendix 2.

The intuition for this result is as follows. The inequality $\alpha_a > \alpha_b$ implies that a transfer by party

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$^6$We note that the first-best value of the venture is precluded by the problem of double moral hazard, for no party can be the unique residual claimant to the investment he makes in such case (Holmström, 1982).
a is less value-decreasing than a transfer by party b. The latter is therefore to be discouraged to a greater extent than is the former. This is achieved by making b’s stake in the venture larger than a’s: \( 1 - \gamma^r > \frac{1}{2} > \gamma^r \).

We argue in what follows that the various clauses included in shareholder agreements are intended to maintain the founding parties’ payoffs in the proportions \( \gamma \) and \( 1 - \gamma \) prescribed by Proposition 1, despite the scope for renegotiation considered in Proposition 2 for example. We initially consider the state \( s_{\overline{TT}} \) where there is no trade buyer who can increase the value of the venture.

3 The state \( s_{\overline{TT}} \): put and call options, pre-emption rights, and tag-along rights

3.1 Put and call options

Initially consider the case where the venture should remain the joint property of the two parties:

\[
V_{ab} (I, T^r, s_{\overline{TT}}) + \alpha_a B (t^r_a) + \alpha_b B (t^r_b) > \max [V_a (I, 0, s_{\overline{TT}}), V_b (I, 0, s_{\overline{TT}})]
\]

(1)

where \( T^r = t^r_a + t^r_b \) denotes the total transfer when the parties’ stakes are \( \gamma^r \) and \( 1 - \gamma^r \). Note that no transfers take place when a single party owns the venture.

We show that a put option held by party a to put a stake \( \gamma - \gamma^r \) to party b at ‘fair’ value, or a call option held by party b to call a stake \( \gamma - \gamma^r \) from party a at fair value, serve to change the parties’ stakes from \( (\gamma, 1 - \gamma) \) to \( (\gamma^r, 1 - \gamma^r) \) while maintaining the parties’ payoffs in the desired proportions \( \gamma \) and \( 1 - \gamma \). We view the ‘fair’ value of the venture as the value of the venture under the conditions that result from the exercise of the option. Shareholder agreements typically include a clause outlining how the venture is to be valued. A popular option is to delegate valuation to an external expert, such as a firm of accountants. Alternatively, the clause may set out a formula for how value is to be determined. Here, we show in Proposition 3 that fair value is equal to
\[ V_{ab}(I, T^r, s_{T^r}) \].

**Proposition 3** Options at fair value serve to minimize ex post transfers without distorting incentives for ex ante investment.

Proof: See Appendix 2. ■

Setting the strike price of the option equal to fair value denies both parties any direct benefit from the exercise of the option. This maintains the parties’ payoffs in the proportions \( \gamma \) and \( 1 - \gamma \). It therefore maintains the parties’ incentives for ex ante investments. Nonetheless, by changing the parties’ stakes from \((\gamma, 1 - \gamma)\) to \((\gamma^r, 1 - \gamma^r)\) prior to the transfers, the exercise of the option makes possible the minimization of ex post transfers.

We note that the choice between a put option granted party \( a \) and a call option granted party \( b \) is not a matter of indifference, for the necessary and sufficient condition for party \( a \) to exercise the put option implies that party \( b \) does not exercise the call option and, conversely, the necessary and sufficient condition for party \( b \) to exercise the call option implies that party \( a \) does not exercise the put option. For example, party \( a \) exercises the put option if and only if

\[
\gamma V_{ab}(I, T^r, s_{T^r}) + \alpha_a B(t_a^r) \\
> \gamma V_{ab}(I, T, s_{T^r}) + \alpha_a B(t_a) \\
+ \beta \left[ V_{ab}(I, T^r, s_{T^r}) + \alpha_a B(t_a^r) + \alpha_b B(t_b^r) \right. \\
- \left. [V_{ab}(I, T, s_{T^r}) + \alpha_a B(t_a) + \alpha_b B(t_b)] \right] \\
\]  

(2)

where \( \beta \) denotes party \( a \)'s bargaining power. But inequality (2) implies that

\[
(1 - \gamma) V_{ab}(I, T^r, s_{T^r}) + \alpha_b B(t_b^r) \\
< (1 - \gamma) V_{ab}(I, T, s_{T^r}) + \alpha_b B(t_b) \\
+ (1 - \beta) \left[ V_{ab}(I, T^r, s_{T^r}) + \alpha_a B(t_a^r) + \alpha_b B(t_b^r) \right. \\
- \left. [V_{ab}(I, T, s_{T^r}) + \alpha_a B(t_a) + \alpha_b B(t_b)] \right]
\]  

8
Thus, a put option will be granted party $a$ when inequality (2) is true, and a call option will be granted party $b$ when it is false.\(^7\)

Now consider the case where inequality (1) is reversed, calling for the venture to be acquired by one or the other of the founding parties. Assume without loss of generality that \(V_a(I, 0, s_{\overline{T}}) < V_b(I, 0, s_{\overline{T}})\). In this case too, we can show that a put option granted party $a$ or a call option granted party $b$ will maintain investment incentives. Either option is for party $a$’s entire stake and is at fair value, which implies a strike price \(\gamma V_b(I, 0, s_{\overline{T}})\).

**Proposition 4** In the case where the minimization of ex post transfers requires the entire venture is to be acquired by a single party following the realization of the state, an option at fair value serves to maintain the parties’ incentives for ex ante investments.

Proof: The proof is similar to that of Proposition 3.\(\blacksquare\)

In this case too, the choice between a put option and a call option is not a matter of indifference. Party $a$ will be granted a put option when the inequality

\[
\gamma V_a(I, 0, s_{\overline{T}}) > \gamma V_{ab}(I, T, s_{\overline{T}}) + a_a B(t_a) + \beta [V_a(I, 0, s_{\overline{T}}) - [V_{ab}(I, T, s_{\overline{T}}) + a_a B(t_a) + d_b B(t_d)]]
\]

is true, and party $b$ will be granted a call option when it is false.

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\(^7\)A sufficient condition for inequality (2) to hold is that \(\beta < \gamma\). The low bargaining power of party $a$ ensures that $a$ wishes to avoid bargaining. This is done by exercising the put option. To establish the sufficiency of the condition \(\beta < \gamma\), assume inequality (2) is false. This implies:

\[
(g - \beta) V_{ab}(I, T, s_{\overline{T}}) + (1 - \beta) a_a B(t_a) - \beta a_b B(t_b)
\]

But the inequality is false as the results $T_T < T$, $t_a > t_a$, and $t_b < t_b$ from the proof of Proposition 2 combine with the assumptions $V_2 < 0$ and $B' > 0$ to imply that each term on the LHS of the inequality is larger than the corresponding term on the RHS.
3.2 Pre-emption rights and tag-along rights

When a founding party wishes to sell his stake in the venture, pre-emption rights grant the remaining party the right to buy the departing party’s stake at fair value. This is so even where the departing party has been offered a higher price for his stake by an outside party.⁸

To motivate the use of pre-emption rights, assume there exists a trade buyer $t_b$ who cannot increase the value of the venture but can extract more value from the venture than can party $a$ for example. Specifically, assume⁹

$$V_{tb}(I, t_{tb} + t_{tb}^{ab}, s_{tb}) + \alpha_{tb}B(t_{tb}) + \alpha_bB(t_{tb})^b < V_{tb}(I, 0, s_{tb})$$

but

$$\gamma V_{tb}(I, t_{tb} + t_{tb}^{ab}, s_{tb}) + \alpha_{tb}B(t_{tb})^b > \gamma V_{ab}(I, t_{a} + t_{ab}^{ab}, s_{ab}) + \alpha_aB(t_{a})^a$$

where $t_{tb}^{ab} \equiv \text{arg max}_{t_{tb}} (1 - \gamma)V_{tb}(I, t_{tb} + t_{tb}^{ab}, s_{tb}) + \alpha_bB(t_{tb})^b$ and $t_{ab}^{ab}$ is defined similarly. Note that $t_{a} > t_{b}^{ab}$ as $\gamma = \frac{1}{2}$, $\alpha_a > \alpha_b$, and $B'' < 0$.

Inequalities (3) and (4) imply that

$$(1 - \gamma)V_{tb}(I, t_{tb} + t_{tb}^{ab}, s_{tb}) + \alpha_bB(t_{tb})^b < (1 - \gamma)V_{ab}(I, t_{a} + t_{ab}^{ab}, s_{ab}) + \alpha_bB(t_{ab})^b$$

⁸Pre-emption rights therefore differ from the right of refusal (see Appendix 1).
⁹In order to simplify the exposition, we neglect the option that one party will have on the stake $\gamma - \gamma''$. This can be shown not to affect our main results.
The preceding inequalities imply that both founding parties will wish to renegotiate the distribution of payoffs rather than have party $a$ sell his stake to the trade buyer. Such renegotiation is ex post efficient, but ex ante inefficient as its distorts the parties’ incentives to invest. We show that pre-emption rights serve to avoid renegotiation, by denying party $a$ the incentive to threaten selling his stake to the trade buyer in the state $s_{tb}$ under the sufficient condition that

$$\gamma V_b (I, 0, s_{tb}) + \beta [V_{ab} (I, t_a + t_b, s_{tb}) + \alpha_a B (t_a) + \alpha_b B (t_{ab})] - V_b (I, 0, s_{tb}) < \gamma V_{ab} (I, t_a + t_b, s_{tb}) + \alpha_a B (t_a)$$

Inequality (6) ensures that party $a$’s payoff, were $b$ to threaten to exercise his pre-emption right in response to $a$’s threat to sell his stake to the trade buyer and the founding parties were to renegotiate, is lower than party $a$’s payoff from refraining from doing so. Party $a$ therefore refrains from threatening to sell his stake.

**Proposition 5** Pre-emption rights serve to deter a party from threatening to sell his stake to a trade buyer who would transfer more value from the venture but would not increase its value.

**Proof:** See Appendix 2.

Tag-along rights, which allow party $b$ to require the trade buyer to buy $b$’s stake on the same terms and conditions as party $a$’s stake, may serve the same role as pre-emption rights. Tag-along rights deny the trade buyer the incentive to engage in transfers, as such rights make the trade buyer’s acquisition of the venture conditional on him being the single owner of the venture. They therefore decrease the price he can offer for party $a$’s stake and thereby diminish the credibility of party $a$’s threat to sell his stake to the trade buyer.

**Proposition 6** Tag-along rights may serve to deter a party from threatening to sell his stake to a trade buyer who would transfer more value from the venture but would not increase its value.
4 The state $s_{tb}$: catch-up clauses, drag-along rights, demand rights, tag-along rights, and pre-emption rights

We now consider the state $s_{tb}$, in which a trade buyer appears who can increase the value of the venture.

4.1 Catch-up clauses

We first consider catch-up clauses. Recall that party $b$ has a call option on part of party $a$’s stake when inequality (1) is true. Thus, in state $s_{tb}$, party $b$ will want to exercise the call prior to the sale of the venture to the trade buyer if he expects the valuation of the strike price not yet to reflect the increase in the value of the venture that will be made possible by the trade sale (perhaps because neither the external valuation expert nor party $a$ are yet aware of the impending trade sale). Exercising the option allows party $b$ to profit from that increase in the proportion $1 - \gamma^r > 1 - \gamma$.

To avoid this outcome, which would distort ex ante investment, catch-up clauses grant party $a$ the right to any additional gain made by party $b$ when exercising the call option on part of party $a$’s stake shortly before selling the venture. This maintains the parties’ payoffs in the desired proportions $\gamma$ and $1 - \gamma$.

4.2 Drag-along rights and demand rights

As is clear from the definition of state $s_{tb}$, both founding parties will gain from the sale of the venture to the trade buyer. Despite such gains, one of the two parties can profit by vetoing the sale of the venture. This is because such veto will lead to bargaining between the parties, as the other party tries to buy the vetoing party’s assent to the value-increasing sale.
To see that at least one party will wish to veto the sale of the venture, consider the conditions necessary for neither party to wish to do so:

\[ \gamma V_{ib} (I, 0, s_{tb}) > \gamma V_{ab} (I, T, s_{tb}) + \alpha_a B(t_a) + \beta [V_{ib} (I, 0, s_{tb}) - [V_{ab} (I, T, s_{tb}) + \alpha_a B(t_a) + \alpha_b B(t_b)]] \] (7)

and

\[ (1 - \gamma) V_{ib} (I, 0, s_{tb}) > (1 - \gamma) V_{ab} (I, T, s_{tb}) + \alpha_b B(t_b) + (1 - \beta) [V_{ib} (I, 0, s_{tb}) - [V_{ab} (I, T, s_{tb}) + \alpha_a B(t_a) + \alpha_b B(t_b)]] \] (8)

The inequalities cannot simultaneously be true, as the sum of their LHS equals that of their RHS. Thus, one party will wish to veto the sale if granted the right to do so.

However, denying both parties the right to veto the sale will not necessarily solve the problem that arises from the unwillingness of one or both founding parties to sell to the trade buyer at the outset. Consider the case where neither holds a veto. Despite this, we can show that one party will hold up the sale, in the expectation of extracting more from the trade buyer by bargaining when the value of the venture is maximized under the trade buyer’s sole ownership. For example, in the case where \(\alpha_a > 0 = \alpha_b = \alpha_{tb}\), party a but not party b will profit from refraining from taking part in the trade sale, for a’s ability to engage in transfers implies that he will be bought out at a premium by
the trade buyer. Formally, we have

\[
\gamma V_{tb}(I, t_a, s_{tb}) + \alpha_a B(t_a) \\
+ [V_{tb}(I, 0, s_{tb}) - [V_{tb}(I, t_a, s_{tb}) + \alpha_a B(t_a)]] \\
= V_{tb}(I, 0, s_{tb}) - (1 - \gamma) V_{tb}(I, t_a, s_{tb}) \\
> V_{tb}(I, 0, s_{tb}) - (1 - \gamma) V_{tb}(I, 0, s_{tb}) \\
= \gamma V_{tb}(I, 0, s_{tb}) 
\]

for party \(a\), and

\[
(1 - \gamma) V_{tb}(I, 0, s_{tb}) \\
+ [V_{tb}(I, 0, s_{tb}) - V_{tb}(I, 0, s_{tb})] \\
= (1 - \gamma) V_{tb}(I, 0, s_{tb})
\]

for party \(b\). In such a case, however, party \(b\) will not be offered \((1 - \gamma) V_{tb}(I, 0, s_{tb})\) by the trade buyer, as the latter’s expectation of bargaining with party \(a\) implies that the most the trade buyer can offer party \(b\) is

\[
V_{tb}(I, 0, s_{tb}) - \left[ \gamma V_{tb}(I, t_a, s_{tb}) + \alpha_a B(t_a) \\
+ [V_{tb}(I, 0, s_{tb}) - [V_{tb}(I, t_a, s_{tb}) + \alpha_a B(t_a)]] \right] \\
< V_{tb}(I, 0, s_{tb}) - \gamma V_{tb}(I, 0, s_{tb}) \\
= (1 - \gamma) V_{tb}(I, 0, s_{tb})
\]

where the inequality is true by the inequality in expression (9). Party \(b\) too will therefore refuse to sell to the trade buyer, and bargaining will occur despite the denial of veto rights to both founding parties. We show in Proposition 7 that drag-along rights, which allow a party selling to a trade buyer to force the other party to join the first party in the trade sale, serve to avoid bargaining.
Proposition 7 Drag-along rights serve to avoid bargaining between the founding parties when the venture is to be sold to a trade buyer.

Proof: It suffices to show that one party will wish to exercise his drag-along rights. But this is immediate from the fact that the two inequalities (7) and (8) cannot simultaneously be false. The party for whom the inequality is true will exercise his drag-along rights.

We now turn to demand rights. These allow a party to force the other party to agree to taking the joint venture public in an IPO. We argue that demand rights are very similar to drag-along rights, in that they are intended to avoid bargaining prior to an IPO.\footnote{This can be formalized by introducing a use $u = \text{ipo}$ and a state $s_{\text{ipo}}$ which are such that: $V_{\text{ipo}}(I, T, s_{\text{ipo}}) > V_u(I, T, s_{\text{ipo}})$ for $u \neq \text{ipo}$.} As with drag-along rights, demand rights deny the parties veto rights. In contrast to drag-along rights, they do not mandate that the parties sell their entire stakes in the IPO. We view this difference as due to the lower ability of parties that hold large stakes in a publicly-quoted company to transfer value from the company, because of the constraints imposed by stock exchanges, regulation, and the law.\footnote{For example, stock exchanges require companies to abide by ‘Continuing Obligations’ that are aimed at protecting outside shareholders.}

4.3 Tag-along rights and pre-emption rights

Tag-along rights are in some ways the mirror image of drag-along rights. The latter grant the party arranging a trade sale the right to force the other party to take part in the trade sale. The former grant the party left out of a trade sale arranged by the other party the right to force the trade buyer to buy its stake.

Section 3.2 has shown that there is a role for tag-along rights when one party threatens to sell his stake to a trade buyer who would not increase the value of the venture but would increase the value of the selling party’s stake through larger transfers from the venture. In this section, we show that there is a role for tag-along rights when one party tries to conspire with a trade buyer who can
increase the value of the venture to exclude the other party from the increase in value. Specifically, assume that

\[ V_{tb}(I, 0, s_{tb}) \]

\[ > V_{tb}(I, t_{tb} + t_{tb}^{th}, s_{tb}) + \alpha_{tb}(t_{tb}) + \alpha_{b}(t_{tb}^{th}) \]

\[ > V_{ab}(I, t_{a} + t_{ab}^{tb}, s_{tb}) + \alpha_{a}(t_{a}) + \alpha_{b}(t_{ab}) \]

and

\[ \gamma V_{tb}(I, t_{tb} + t_{tb}^{th}, s_{tb}) + \alpha_{tb}(t_{tb}) \]

\[ > \gamma V_{tb}(I, 0, s_{tb}) \]

\[ > \gamma V_{ab}(I, t_{a} + t_{ab}^{tb}, s_{tb}) + \alpha_{a}(t_{a}) \]

but

\[ (1 - \gamma) V_{tb}(I, t_{tb} + t_{tb}^{th}, s_{tb}) + \alpha_{b}(t_{tb}^{th}) \]

\[ < (1 - \gamma) V_{tb}(I, 0, s_{tb}) \]

\[ < (1 - \gamma) V_{ab}(I, t_{a} + t_{ab}^{tb}, s_{tb}) + \alpha_{b}(t_{ab}) \]

Under these circumstances, it is clear that party \( a \) would like to conspire with the trade buyer to have the trade buyer buy party \( a \)'s stake at the following price:

\[ \gamma V_{tb}(I, t_{tb} + t_{tb}^{th}, s_{tb}) + \alpha_{tb}(t_{tb}) > \gamma V_{tb}(I, 0, s_{tb}) \]

Party \( a \)'s gain is at the expense of party \( b \), whose payoff after negotiating with the trade buyer
for the latter to buy the former’s stake is

\[
(1 - \gamma) V_{tb} (I, t_{tb} + t^b, s_{tb}) + \alpha_b B (t^b) \\
+ [V_{tb} (I, 0, s_{tb}) - [V_{tb} (I, t_{tb} + t^b, s_{tb}) + \alpha_{tb} B (t_{tb}) + \alpha_b B (t^b)]]
\]

\[
= V_{tb} (I, 0, s_{tb}) - [\gamma V_{tb} (I, t_{tb} + t^b, s_{tb}) + \alpha_{tb} B (t_{tb})]
\]

\[
< V_{tb} (I, 0, s_{tb}) - [\gamma V_{tb} (I, 0, s_{tb})] \tag{10}
\]

\[
= (1 - \gamma) V_{tb} (I, 0, s_{tb})
\]

The founding parties’ payoffs are thereby altered from the desired proportions \(\gamma\) and \(1 - \gamma\). We show in Proposition 8 that tag-along rights granted party \(b\) serve to maintain the parties’ payoffs in these proportions.

**Proposition 8** Tag-along rights preclude a founding party from conspiring with a trade buyer to exclude the other founding party from sharing in the increase in value made possible by the sale of the venture to the trade buyer in the desired proportions \(\gamma\) and \(1 - \gamma\).

Proof: It suffices to show that party \(b\) will exercise his tag-along rights, for the obligation for the trade buyer to buy the parties’ stakes on the same terms and conditions in that case implies that the parties will receive the desired \(\gamma V_{tb} (I, 0, s_{tb})\) and \((1 - \gamma) V_{tb} (I, 0, s_{tb})\). But that party \(b\) will exercise his drag-along rights is immediate from inequality (10).

We have seen in Section 3.2 that pre-emption rights and drag-along rights are to some extent substitutes in the case where one party threatens to sell the venture to a trade buyer who would extract more value from the venture but would not increase its value. These rights are also substitutes in the present case. In particular, if

\[
\gamma V_b (I, 0, s_{tb}) + \beta [V_{tb} (I, 0, s_{tb}) - V_b (I, 0, s_{tb})] < \gamma V_{tb} (I, 0, s_{tb})
\]

then party \(b\)’s pre-emption rights can be shown to preclude party \(a\) from conspiring with the trade
buyer. If the reverse inequality holds, then pre-emption rights fail to do so.

5 Continuation and termination in alliances

We briefly consider the issue of whether to continue or terminate an alliance. An alliance is a form of joint undertaking that often has a pre-specified finite life, after which it is terminated unless the party with the option to extend its life for an additional period chooses to do so. We argue in this section that the purpose of this option is to avoid renegotiation.

We modify the model of the preceding sections as follows. We denote the value of the alliance $V_c(i_a i_b, s)$ when continued and $V_t(i_a i_b, s)$ when terminated. We neglect ex post transfers for simplicity but without loss of generality. Let $s_c$ denote the state of the world in which the alliance should be continued and $s_t$ denote that in which it should be terminated. Thus,

$$V_c(i_a i_b, s_c) > V_t(i_a i_b, s_c)$$

and

$$V_c(i_a i_b, s_t) < V_t(i_a i_b, s_t)$$

As in Section 2, we can show that equal stakes maximize the value of the alliance, subject to the constraints imposed by the problem of double moral hazard: $\gamma = \frac{1}{2} = 1 - \gamma$. To motivate the use of the option to extend the life of the alliance, consider state $s_c$ in which the alliance should be continued and each party has payoff $\gamma V_c(i_a i_b, s_c)$. Can a party, say party $a$, profit from threatening not to agree to the continuation of the alliance for the purpose of bargaining with party $b$? Party

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12 We thank Josh Lerner for encouraging us to do so.
\[ \gamma V_t (i_a i_b, s_c) + \beta [V_c (i_a i_b, s_c) - V_t (i_a i_b, s_c)] \]

\[ = \beta V_c (i_a i_b, s_c) - (\beta - \gamma) V_t (i_a i_b, s_c) \]

\[ > \gamma V_c (i_a i_b, s_c) \]

for \( \beta > \gamma \). This problem cannot be solved by specifying that the life of the alliance be infinite, for party a would then threaten not to agree to the termination of the alliance in state \( s_t \). The problem, of course, is that a change in the status of the alliance that requires party a’s agreement provides a with an opportunity to exploit his greater bargaining power.

The problem can be solved by granting party b the option to extend the life of the alliance, for such an option dispenses party b from seeking party a’s approval. The option will not be exploited by party b, for his lower bargaining power implies that he has nothing to gain from bargaining. Indeed, consider an attempt by party b to threaten continuing the alliance in state \( s_t \). His payoff from doing so is

\[ (1 - \gamma) V_c (i_a i_b, s_t) + (1 - \beta) [V_t (i_a i_b, s_t) - V_c (i_a i_b, s_t)] \]

\[ = (1 - \beta) V_t (i_a i_b, s_t) + (\beta - \gamma) V_c (i_a i_b, s_t) \]

\[ < (1 - \gamma) V_t (i_a i_b, s_t) \]

as \( V_c (i_a i_b, s_t) < V_t (i_a i_b, s_t) \). Party b will therefore not exploit his option.

6 Conclusion

We have presented an explanation for a number of key clauses that often appear in shareholder agreements, such as those between partners in a joint venture and between a venture capitalist and an entrepreneur. In the presence of a problem of dynamic, double moral hazard, the clauses preserve
the parties’ incentives to make ex ante investments and they minimize ex post transfers.

Much of the analysis has revolved around the idea that clauses are used to avoid renegotiation. Yet, renegotiation often occurs in practice (Lerner and Tsai, 2000). We ascribe such renegotiation to three factors. Renegotiation is likely to arise in the presence of financing constraints, in the presence of asymmetric information, and when the conditions that must hold for pre-emption rights and tag-along rights to be effective are not true.

Consider financing constraints first. Recall that our model requires the parties to own equal shares of the joint undertaking. But wealth constraints on one or the other party may mean that one party owns a larger share of the venture at the outset. In such case, should the wealth constraint be relaxed at some point in the future, perhaps as a result of a change in the availability of external finance, one would expect the parties to renegotiate their shares.13

Now consider asymmetric information. Our model has assumed that, on realization of the state, there was no asymmetry of information among the parties. Yet, at least in the case where one party is to buy out the other, it is likely that each party has better knowledge of the value of the venture to itself than does the other party. Under such circumstances, the party that has been granted the put option may mistakenly believe that the value of the venture is higher for the other party than it is for itself, and exercise his put option when he should not. Renegotiation would occur in such case.

Finally, consider the conditions that must hold for pre-emption rights and tag-along rights to be effective. Should these condition not be satisfied, it is likely that the corresponding clauses will fail in their intended purpose of avoiding renegotiation. They may not even be included in the contract.

We acknowledge the importance of the preceding considerations and their potential to explain contract clauses that we have not explained, such as the right of first refusal. We leave these considerations for future research.

13Lerner and Tsai (2000) document such patterns in biotechnology alliances.
Appendix 1: An overview of shareholder agreements

Standard shareholder agreements typically contain the following articles or groups of articles (Bernstein, 1988; Freedman, 1994; Martel, 1991; Stedman and Jones, 1990):

- Termination of prior agreements between some or all shareholders regarding the organization and affairs of the company, as well as warranties and covenants specifying that all shares are free and clear of all claims.

- Provision of control: Designation of the rights and duties of the shareholders in the management of the company, and requirement of prior unanimous consent for major decisions such as the declaration of any dividend and the issuance or sale of shares.

- Restrictions on the transfer of shares: The shareholders commit not to sell, pledge, or charge their shares except with the prior written consent of all other shareholders.

- Survivorship arrangements: Upon the death of any shareholder, the personal representatives of the deceased shall sell the shares of the deceased to the company, typically at a price specified in the article on valuation. Life insurance policies will be issued to the benefit of the shareholders to ensure that this article can be enforced.

- Valuation: The ‘fair’ value of the shares is generally determined by an external expert, or it is based on a previously agreed upon valuation formula.

- Right of first refusal: A shareholder offered to sell his shares to an outside investor at some price is required to offer his shares to the other shareholders at the same price. If the other shareholders decline, the first shareholder is free to sell his shares to the outside investor.

- Pre-emption rights: A shareholder wishing to sell his stake in the company is required to offer his shares to the other shareholders. Pre-emption rights can take several forms. In the extreme, selling the shares to an outside investor is actually prohibited.

- Put options: A shareholder is granted put options on the shares held by the other shareholders. The strike price is generally the ‘fair’ value of the shares.
• Call options: Similar to put options.

• Catch up clauses: When a shareholder exercises a call option, the selling shareholder maintains a claim on part of the payoff subsequently realized by the first shareholder in a trade sale or an IPO.

• Drag-along rights: In case a shareholder sells his stake to an outside investor, drag-along rights grant the investor the right to buy out the other shareholders’ stakes at the same price and on the same terms as the first shareholder’s stake. Drag-along rights can be viewed as conditional call options granted the outside investor.

• Tag-along rights (or piggy-back rights, or co-sale agreements): In case a shareholder sells his stake to an outside investor, tag-along rights grant the other shareholders the right to require the outside investor to buy these shareholders’ stakes at the same price and on the same terms as the first shareholder’s stake. Tag-along rights can be viewed as conditional put options granted all shareholders.

• Demand rights (or initial public offering clauses): Shareholders agree in advance the circumstances in which they will take the company public. Demand rights ensure that the company will be taken public once a prespecified level of profit is achieved, or when the company has a specific need for outside finance. Demand rights may require all shareholders to participate in the offering.

• Non-competition: Each and every shareholder undertakes not to compete with the venture.

• Dispute resolution and arbitration: The shareholders agree to follow a specified procedure to resolve disputes. The procedure may specify the appointment of an arbitrator.
Appendix 2: Proofs

**Proof of Proposition 1:** \( \gamma \) is the solution to the problem

\[
\max_{\gamma} \, V_u(i_a i_b, 0, s) - i_a - i_b
\]

where

\[
i_a = \arg \max_{\hat{i}_a} \gamma V_u\left(\hat{i}_a i_b, 0, s\right) - \hat{i}_a
\]

and

\[
i_b = \arg \max_{\hat{i}_b} (1 - \gamma) V_u\left(\hat{i}_a \hat{i}_b, 0, s\right) - \hat{i}_b
\]

The corresponding first-order conditions are

\[
V_{u,1}(i_a i_b, 0, s) \left[ \frac{\partial i_a}{\partial \gamma} + i_a \frac{\partial i_b}{\partial \gamma} \right] - \frac{\partial i_a}{\partial \gamma} - \frac{\partial i_b}{\partial \gamma} = 0 \quad (11)
\]

\[
i_b \gamma V_{u,1}(i_a i_b, 0, s) - 1 = 0 \quad (12)
\]

and

\[
i_a(1 - \gamma) V_{u,1}(i_a i_b, 0, s) - 1 = 0 \quad (13)
\]

Equations (12) and (13) imply

\[
\gamma i_b = (1 - \gamma) i_a \quad (14)
\]
Using equations (12), (13), and (14), we can rewrite equation (11) as

\[(1 - \gamma)^2 \frac{\partial i_a}{\partial \gamma} + \gamma^2 \frac{\partial i_b}{\partial \gamma} = 0\]  \hspace{1cm} (15)

Totally differentiating equation (14) with respect to \(\gamma\), we obtain

\[(1 - \gamma) \frac{\partial i_a}{\partial \gamma} = \gamma \frac{\partial i_b}{\partial \gamma} + i_a + i_b\]  \hspace{1cm} (16)

Substituting equation (16) into equation (15), we have

\[\frac{\partial i_b}{\partial \gamma} = -\frac{i_b}{\gamma}\]  \hspace{1cm} (17)

Substituting equation (17) into equation (16), we obtain

\[\frac{\partial i_a}{\partial \gamma} = \frac{i_a}{1 - \gamma}\]  \hspace{1cm} (18)

We conjecture a solution \(\gamma = \frac{1}{2}\) and denote the corresponding value of the venture

\[V = V_u (i^2, 0, s) - 2i\]

where we have used equation (14) to write \(i_a = i_b \equiv i\).

To show that \(\gamma = \frac{1}{2}\) is a maximum, we compute the value of the venture corresponding to \(\gamma_\delta = \frac{1}{2} + \delta\). Let \(i_{a,\delta}\) and \(i_{b,\delta}\) denote the corresponding investments made by the parties. From equations (18) and (17) we have

\[i_{a,\delta} = i_a + \delta \frac{\partial i_a}{\partial \gamma} = i_a [1 + 2\delta] = i [1 + 2\delta]\]

\[i_{b,\delta} = i_b + \delta \frac{\partial i_b}{\partial \gamma} = i_b [1 - 2\delta] = i [1 - 2\delta]\]
The corresponding value of the venture equals

\[ V_\delta = V_a(i_{a,\delta}i_{b,\delta},0,s) - i_{a,\delta} - i_{b,\delta} \]
\[ = V_a(i^2[1 + 2\delta][1 - 2\delta],0,s) - 2i \]
\[ = V_a(i^2[1 - 4\delta^2],0,s) - 2i \]
\[ < V \]

where the inequality is true by the assumption that \( V_{a,1} > 0 \).

**Proof of Proposition 2:** The stake \( \gamma^r \) is the solution to the problem

\[ \text{Max} \gamma^r V_{ab}(I_t^r + t_{b}^r, s_{\gamma^r}) + \alpha_a B(t_{a}^r) + \alpha_b B(t_{b}^r) \]

where

\[ t_{a}^r = \arg \max_{\hat{t}_a} \gamma^r V_{ab}(I, \hat{t}_a + t_{b}^r, s_{\gamma^r}) + \alpha_a B(\hat{t}_a) \]

and

\[ t_{b}^r = \arg \max_{\hat{t}_b} (1 - \gamma^r) V_{ab}(I, t_{a}^r + \hat{t}_b, s_{\gamma^r}) + \alpha_b B(\hat{t}_b) \]

The corresponding first-order conditions are

\[ V_2(I, t_{a}^r + t_{b}^r, s_{\gamma^r}) \left[ \frac{\partial t_{a}^r}{\partial \gamma^r} + \frac{\partial t_{b}^r}{\partial \gamma^r} \right] + \alpha_a B'(t_{a}^r) \frac{\partial t_{a}^r}{\partial \gamma^r} + \alpha_b B'(t_{b}^r) \frac{\partial t_{b}^r}{\partial \gamma^r} = 0 \] (19)

\[ \gamma^r V_2(I, t_{a}^r + t_{b}^r, s_{\gamma^r}) + \alpha_a B'(t_{a}^r) = 0 \] (20)
and

\[(1 - \gamma^r) V_2 (I, t_a^r + t_b^r, s_{TB}) + \alpha_b B'(t_b^r) = 0 \tag{21}\]

From equations (20) and (21), we obtain

\[
\frac{\partial t_a^r}{\partial \gamma^r} = -\frac{V_2 [V_{22} + \alpha_b B'']}{\gamma^r V_2 2a_b B'' + \alpha_a B'' [(1 - \gamma^r) V_{22} + \alpha_b B'']} < 0
\]

and

\[
\frac{\partial t_b^r}{\partial \gamma^r} = \frac{V_2 [V_{22} + \alpha_a B'']}{\gamma^r V_2 2a_b B'' + \alpha_a B'' [(1 - \gamma^r) V_{22} + \alpha_b B'']} > 0
\]

which implies that \(\left| \frac{\partial t_a^r}{\partial \gamma^r} \right| > \left| \frac{\partial t_b^r}{\partial \gamma^r} \right|\) as \(\alpha_a > \alpha_b\). Substituting equations (20) and (21) into equation (19), we have

\[(1 - \gamma^r) \frac{\partial t_a^r}{\partial \gamma^r} + \gamma^r \frac{\partial t_b^r}{\partial \gamma^r} = 0\]

\[\Leftrightarrow \frac{\gamma^r}{1 - \gamma^r} = -\frac{\partial t_a^r}{\partial \gamma^r} \frac{\partial t_b^r}{\partial \gamma^r} < 1\]

\[\Rightarrow \gamma^r < \frac{1}{2} = \gamma\]

The preceding implies that as the parties’ stakes change from \((\gamma, 1 - \gamma)\) to \((\gamma^r, 1 - \gamma^r)\), \(t_a\) increases to \(t_a^r\), \(t_b\) decreases to \(t_b^r\), and \(T = t_a + t_b\) decreases to \(T^r = t_a^r + t_b^r\). \(\blacksquare\)

**Proof of Proposition 3:** Consider the case where party \(a\) has been granted a put option at fair value on the stake \(\gamma - \gamma^r\).\(^{14}\) Let \(F\) denote the fair value of the venture under the conditions

\(^{14}\)The case where party \(b\) has been granted a call option is similar.
that result from the exercise of the option. Following the realization of the state, the parties choose ex post transfers so as to

\[
\max_{\hat{t}_a} \gamma V_{ab} (I, \hat{t}_a + t_b, \hat{s}_{TB}) + [-(\gamma - \gamma^r) V_{ab} (I, \hat{t}_a + t_b, s_{TB}) + (\gamma - \gamma^r) F] + \alpha_a B (\hat{t}_a)
\]

and

\[
\max_{\hat{t}_b} (1 - \gamma^r) V_{ab} (I, t_a + \hat{t}_b, s_{TB}) - (\gamma - \gamma^r) F + \alpha_b B (\hat{t}_b)
\]

Clearly, parties \( a \) and \( b \) will engage in the transfers \( t^r_a \) and \( t^r_b \), as desired. The fair value \( F \) of the venture under the conditions that result from the exercise of the option therefore equals \( V_{ab} (I, T^r, s_{TB}) \). The strike price equals \( (\gamma - \gamma^r) V_{ab} (I, T^r, s_{TB}) \), thereby ensuring that party \( a \) does indeed exercise the put option.

The preceding implies that, when making the ex ante investments, the parties’ payoffs conditional on the state \( s_{TB} \) being realized are

\[
\gamma^r V_{ab} (I, T^r, s_{TB}) + (\gamma - \gamma^r) F + \alpha_a B (t^r_a)
\]

for party \( a \) and \( (1 - \gamma) V_{ab} (I, T^r, s_{TB}) + \alpha_b B (t^r_b) \) for party \( b \). The parties’ payoffs have been maintained in the desired proportions \( \gamma \) and \( 1 - \gamma \).

**Proof of Proposition 5:** Inequality (6) ensures that party \( a \) will not attempt to sell his stake to the trade buyer if he expects party \( b \) to threaten to exercise his pre-emption rights. It remains to
show that party $b$ will indeed threaten to exercise these rights. His payoff if he does is

$$(1 - \gamma) V_b (I, 0, s_{ab})$$

$$+ (1 - \beta) \left[ V_{ab} (I, t_\alpha + t_{ab}^{tb}, s_{ab}) + \alpha_a B (t_\alpha) + \alpha_b B (t_{ab}^{tb}) - V_b (I, 0, s_{ab}) \right]$$

$$> (1 - \gamma) V_{ab} (I, t_\alpha + t_{ab}^{tb}, s_{ab}) + \alpha_b B (t_{ab}^{tb})$$

where the inequality is true from inequality (6). His payoff if he does not and renegotiates with party $a$ is

$$(1 - \gamma) V_{ib} (I, t_{ib} + t_{ib}^{lb}, s_{ib}) + \alpha_b B (t_{ib}^{lb})$$

$$+ (1 - \beta) \left[ V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_a B (t_\alpha) + \alpha_b B (t_{ab}^{lb}) - V_{ib} (I, t_{ib} + t_{ib}^{lb}, s_{ib}) \right]$$

$$< (1 - \gamma) V_{ib} (I, t_{ib} + t_{ib}^{lb}, s_{ib}) + \alpha_b B (t_{ib}^{lb})$$

$$+ \left[ V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_a B (t_\alpha) + \alpha_b B (t_{ab}^{lb}) - V_{ib} (I, t_{ib} + t_{ib}^{lb}, s_{ib}) + \alpha_{ib} B (t_{ib}) \right]$$

$$= V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_a B (t_\alpha) + \alpha_b B (t_{ab}^{lb})$$

$$- \left[ \gamma V_{ib} (I, t_{ib} + t_{ib}^{lb}, s_{ib}) + \alpha_{ib} B (t_{ib}) \right]$$

$$< V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_a B (t_\alpha) + \alpha_b B (t_{ab}^{lb})$$

$$- \left[ \gamma V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_a B (t_\alpha) \right]$$

$$= (1 - \gamma) V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_b B (t_{ab}^{lb})$$

where the second inequality is true by inequality (4). \[ \blacksquare \]

**Proof of Proposition 6:** Let $P$ denote the price that the trade buyer would pay for the venture. This price must be such that

$$\gamma P > \gamma V_{ab} (I, t_\alpha + t_{ab}^{lb}, s_{ib}) + \alpha_a B (t_\alpha)$$

(22)
Both inequalities are necessary for party a’s threat to sell his stake to the trade buyer tb to be credible. Party a would not wish to sell his stake if inequality (22) were false, and the trade buyer tb would not wish to buy the venture if inequality (23) were false.

Combined with inequality (3), inequalities (22) and (23) imply the necessary condition

\[
V_{ab} (I, t_a + t_{ab}^b, s_{tb}) + \alpha_a B(t_a) > V_{ab} (I, t_a + t_{ab}^b, s_{tb}) + \alpha_b B(t_{ab}^b)
\]

But this condition is false as \( \frac{\alpha_a}{\alpha_b} = 2\alpha_a > \alpha_a + \alpha_b \) and \( B(t_a) > B(t_{ab}^b) \) as \( t_a > t_{ab}^b \). It is therefore impossible for \( P \) to satisfy inequalities (22) and (23) simultaneously.

Note that tag-along rights do not always succeed in deterring the sale to the trade buyer. For example, if

\[
V_{ib} (I, 0, s_{tb}) \approx V_{ab} (I, t_a + t_{ab}^b, s_{tb}) + \alpha_a B(t_a) + \alpha_b B(t_{ab}^b)
\]

then party a’s tag-along rights can be shown to fail in deterring party b from threatening to sell his stake to the trade buyer tb. To see this, let \( P = V_{ib} (I, 0, s_{tb}) \) and note that

\[
(1 - \gamma) P \approx (1 - \gamma) V_{ab} (I, t_a + t_{ab}^b, s_{tb}) + (1 - \gamma) \alpha_a B(t_a) + (1 - \gamma) \alpha_b B(t_{ab}^b)
\]

\[
> (1 - \gamma) V_{ab} (I, t_a + t_{ab}^b, s_{tb}) + (1 - \gamma) \alpha_b B(t_{ab}^b) + (1 - \gamma) \alpha_b B(t_{ab}^b)
\]

\[
= (1 - \gamma) V_{ab} (I, t_a + t_{ab}^b, s_{tb}) + \alpha_b B(t_{ab}^b)
\]

Note that the exercise of the option on the stake \( \gamma - \gamma^r \) does not invalidate this conclusion, as \( \gamma^r < \gamma, \ t_a^r > t_a \), and \( t_{ab}^{br} < t_{ab}^b \) from the proof of Proposition 2.
where the inequality is true as $\alpha_n > \alpha_b$ and $B(t_a) > B(t_b^b)$ as $t_a > t_{b}^b$ and the second equality is true as $\gamma = \frac{1}{2}$. ■
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